Assessment of near-surface ground temperature profiles for optimal placement of a thermoelectric device

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1. Introduction and previous work

With the drive towards energy efficiency in all aspects of society, thermoelectric devices are becoming more and more popular as energy sources, particularly those which are driven by small temperature differences occurring in nature [1,2]. While these small temperature differences do not produce power sufficient to drive large equipment, they may be adequate for applications such as remote sensors powered by ground-source heat engines. Additionally, if such a device were powered by the cyclic temperature difference between the ground and the ambient air, the thermoelectric apparatus could provide power even during hours of darkness, a limitation of most solar systems.

Operation of any such thermoelectric device will be most efficient when placed at the optimal depth, thereby maximizing the power produced during a daily or seasonal quasi-steady cycle. Stevens [3] considered the theoretical performance of a thermoelectric device subjected to a sinusoidal surface temperature boundary condition (Fig. 1), where the temperature distribution in the ground is the solution to the one-dimensional, unsteady heat equation with boundary conditions:

\[ \frac{\partial^2 T}{\partial x^2} = \frac{1}{2} \frac{\partial T}{\partial t} \]
\[ T(x = 0, t) = T_m + \Delta T \cos \left( \frac{2\pi t}{t_0} \right) \]
\[ T(x = \infty, t) = T_m \]

Initial conditions are not required since only the quasisteady solution is desired. Employing the following nondimensionalization:

\[ t' = \frac{2\pi t}{t_0} \quad x' = \sqrt{\frac{\pi}{2t_0}} \quad T'(x', t') = \frac{T(x, t) - T_m}{\Delta T} \]

the dimensionless form of the heat equation and associated boundary conditions are obtained:

\[ \frac{\partial^2 T}{\partial x'^2} = 2 \frac{\partial T}{\partial t'} \]
\[ T'(x' = \infty, t') = 0 \]
\[ T'(x' = 0, t') = \cos(t') \]

Complex combination [4] provides the nondimensional temperature distribution which is the solution to Eq. (3):

\[ T'(x', t') = e^{-x'} \cos(t' - x') \]

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Note that the first factor on the right-hand side is depth-dependent, and the trigonometric factor is temporally-dependent; additionally, there is a depth-dependent phase shift of $x'$ in the temporal portion.

A dimensionless air–ground temperature difference may be defined as

\[ \Delta T_{AG} = \frac{T(0, t) - T(x, t)}{\Delta T} \]

which, for the surface boundary condition imposed by Eq. (3), results in

\[ \Delta T_{AG} = \cos(t') - e^{-x'} \cos(t' - x') \]
It should be noted that, for the analytical case considered by Stevens [1], the surface and ambient temperatures were equal for any time, corresponding to the case of an infinitely-large convection heat transfer coefficient. For a ground-source heat engine, the sign of $\Delta T_{AG}$ is immaterial (i.e., the system generates electricity in either case, with only the polarity of the electricity changing). In fact, it can be shown [1] that, for a particular depth, the energy generated is proportional to $(\Delta T_{AG})^2$, and that the energy generated over a complete cycle is proportional to

$$f(x^*) = \int_0^{2\pi} (\Delta T_{AG})^2 dt$$

Setting the derivative of $f(x^*) = 0$, it is found that the optimal location for placement of the lower thermal reservoir is at a location of $x^* = 2.28$. Additionally, for the boundary condition of Eq. (3), it may be shown that

$$\Delta T_{AG} = A \cos(t^* - \phi)$$

where the energy amplification magnitude and associated phase shift are, respectively,

$$A = \sqrt{1 - 2e^{-x^*} \cos(x^*) + e^{-2x^*}}$$

and

$$\phi = \tan^{-1}\left(\frac{-e^{-x^*} \sin(x^*)}{1 - e^{-x^*} \cos(x^*)}\right)$$

At the optimum value of $x^* = 2.28$, the corresponding value of $A$ is 1.07; this implies that the optimum depth will produce 7% more power than would have been obtained with the lower thermal reservoir located at an infinite depth. This result is provided graphically in Fig. 2. In the limit of $x^* \rightarrow \infty$, $A \rightarrow 1$, becoming horizontally tangent in Fig. 2.

### 2. Objective

During the testing of a thermoelectric device [5], several weeks of ground temperature and performance data were collected by placing thermocouples at various depths in the ground in the proximity of the thermoelectric device. The data were taken in Colorado Springs, Colorado, USA (38° 49' N/104° 43' W, elevation 6700 ft or 2042 m) during the timeframes and at the depths indicated in Table 1.

Fig. 3 shows a sample of smoothed temperature data. Clearly, neither the air temperature nor the temperature at a near-surface depth of 0.5 in. (1.3 cm) remotely resembles the sinusoidal boundary condition assumed in the analytical approach. However, direct analysis of the measured temperature data will provide an optimal placement depth and amplification factor that will be compared to the analytical result.

### 3. Data collection and analysis

Although periodic in nature, neither the air nor near-surface temperature distribution shown in Fig. 3 resembles the cosine variation used by Stevens [3] in his analytical model. A somewhat more accurate representation of the temperature profile might be obtained with a saw tooth-shaped distribution, or perhaps a step function. Therefore, it was prudent to investigate how these distributions would affect the location of the optimal position and its corresponding value of $A$. A finite-difference model was constructed to evaluate how these parameters would be affected by different surface temperature distributions.

### Table 1

<table>
<thead>
<tr>
<th>Timeframe</th>
<th>Thermocouple locations (in/cm)</th>
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<tbody>
<tr>
<td>21 October–2 November 2005</td>
<td>0.5/1.3, 4/10.2, 9/22.9, 14/35.6</td>
</tr>
<tr>
<td>23 May–1 June 2006</td>
<td>0.5/1.3, 1/2.5, 9/22.9, 14/35.6</td>
</tr>
<tr>
<td>3–15 June 2006</td>
<td>0.5/1.3, 1/2.5, 9/22.9, 14/35.6</td>
</tr>
</tbody>
</table>
The finite-difference model employed a nondimensional time step of $\Delta t^* = 0.005$, a nondimensional element length of $\Delta x^* = 0.125$, and a “deep” boundary condition of $T(10, t^*) = 0$ (see Eqs. (2) and (3)); an initial condition of $T(x^*, 0) = 0$ throughout the computational domain was employed. In order for the model to achieve a quasi-steady state, 3 cycles were run for each boundary condition.

The model was first verified against the cosine surface temperature boundary condition; the results could then be compared to the exact analytical results of $A_{\text{optimum}} = 1.07$ and $x_{\text{optimum}}^* = 2.28$. Table 2 shows that the results for the cosine boundary condition are in excellent agreement with the analytical test case; for the saw tooth and step boundary conditions, the location and amplification magnitude for the optimal point are not substantially different. From these three radically different boundary conditions, it can be concluded that the shape alone of a periodic temperature boundary condition has little effect on optimal depth and amplification. Accordingly, the cosine surface temperature distribution employed by Stevens [3] will be used as a basis of comparison for the experimental data analyzed in the present work.

The site at which data were taken was a reasonably sheltered area, not subjected to the full force of the prevailing winds. Additionally, detailed data were not available for humidity and cloud cover in the immediate vicinity of the site. The soil at the test site was best described as a type of Moody silt, whose thermophysical properties are provided in Table 3.

The measured temperature data were analyzed using the same nondimensionalization shown in Eqs. (2) and (5). While it was originally thought that curve fits through the four thermocouple temperatures could be used, a check using four points from an analytical temperature distribution (Eq. (4), the analytical solution to Eq. (1), with $T_m = 50^\circ F$ ($10.0^\circ C$), $\Delta T = 20^\circ F$ ($6.7^\circ C$), and a period of $t_0 = 24$ h) for the curve fit showed that a cubic spline could not faithfully replicate the thermocouple temperature distribution (Fig. 4).

Instead, it was decided that the experimental energy amplification, $A'$, would be calculated at each of the data points, using each day as a 24-h cycle. The deepest thermocouple location was 14 in. (35.6 cm) (a nondimensional depth of $x^* = 3.13$), but it can be seen from Fig. 3 that the temperature at that depth was not steady. For the test site, the undisturbed ground temperature is given as $51^\circ F$ ($10.6^\circ C$), and is normally specified at a depth of 20 ft (6.1 m) ($x^* = 53.7$); this temperature is a year-round constant, but there may also exist daily and seasonal averages which change very slowly about the year-round average [4,6]. The exponential term of Eq. (4) shows that the temperature variation falls off quickly with depth; a variation of one percent of the surface temperature variation correlates to an approximate nondimensional penetration depth of $x^* = 4.6$. Consequently, it was decided that the average temperature at the deepest (14 in. (35.6 cm), $x^* = 3.13$) thermocouple for each day would be used as the undisturbed

<table>
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<th>Table 2</th>
<th>Variation of optimum location for various surface temperature distributions.</th>
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<tr>
<td>Surface temperature distribution</td>
<td>$x_{\text{optimum}}^*$</td>
</tr>
<tr>
<td>Cosine</td>
<td>2.26</td>
</tr>
<tr>
<td>Saw tooth</td>
<td>2.38</td>
</tr>
<tr>
<td>Step</td>
<td>2.39</td>
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<tr>
<th>Table 3</th>
<th>Soil thermophysical properties [7].</th>
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<tr>
<td>Property</td>
<td>Symbol</td>
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<tr>
<td>Thermal conductivity</td>
<td>$k$</td>
</tr>
<tr>
<td>Specific heat</td>
<td>$c$</td>
</tr>
<tr>
<td>Density</td>
<td>$\rho$</td>
</tr>
<tr>
<td>Thermal diffusivity</td>
<td>$\alpha$</td>
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Fig. 3. Sample of smoothed air and thermocouple temperature data for October–November 2005.

Fig. 4. Comparison of cubic spline curve fit vs. exact temperature distribution.
ground temperature at a depth of 24 in. (61.0 cm) \( (x^* = 5.37) \). This resulted in a total of five points to use in constructing the energy amplification curve: four resulting from thermocouple data, and a fifth at the assumed position of the undisturbed ground temperature.

By varying the depth of the undisturbed ground temperature in an analytical test case, the results were not significantly affected unless it was placed very close to the deepest thermocouple. The results of this test are illustrated in Fig. 5, where the analytical energy amplification curve obtained from Eq. (9) for the cosine surface temperature boundary condition is plotted; five sampling points were taken at the thermocouple locations and the assumed position for the undisturbed ground temperature \((x^*)\). These five points were then used to construct the cubic spline approximation shown—agreement with Eq. (9) was excellent.

For data reduction (in lieu of Eq. (7)), the energy amplitude magnification, \( A' \), for each thermocouple location is calculated as

\[
A' = \frac{\text{energy for thermocouple location}}{\text{energy for 24 in. (61.0 cm) depth}} = \sqrt[ \frac{\sum (T_{\text{thermocouple}} - T_{\text{air}})^2}{\sum (T_{24\text{ in. position}} - T_{\text{air}})^2} }
\]

where the summations are conducted over all points between the assumed position of undisturbed ground temperature and the surface. This may be thought of as a “normalized, root-mean-square temperature difference.”

Once the value of the energy amplitude magnification, \( A' \), was calculated for each thermocouple location for each 24-h period, there were a total of five points: one for each of the four thermocouples, along with a value of \( A' = 1 \) at the 24 in. (61.0 cm) location of the undisturbed ground temperature. A cubic spline curve fit was performed on the values of \( A' \) vs. \( x^* \). The shape of the \( A \) vs. \( x^* \) curve in Fig. 2 suggests that a free-spline end condition should be used at the shallowest thermocouple location, while a zero-slope end condition is appropriate at the location of the undisturbed ground temperature.

Employing the above rationale, samples of the resulting reduced data for the 21 October–2 November 2005 are shown in Fig. 6. A brief description of each plot follows:

- a. Average temperatures for each thermocouple and air for each day.
- b. Optimal nondimensional depth and maximum energy amplification factor for each day.
- c. The difference between the maximum and minimum temperatures for each thermocouple and the air for each day.
- d. The energy amplification curve for each day. Nondimensionalized thermocouple locations are indicated by vertical lines.
4. Data interpretation

Although sample results are presented only for the 21 October–2 November 2005 timeframe, the results for all three time periods are similar. Based upon the sensitivity analysis performed on different surface temperature distributions with the finite-difference simulation, the saw tooth air and near-surface temperature distributions of Fig. 3 should not have been a significant factor. It is interesting to note that temperature swings (Fig. 6c) become successively smaller with increasing depth. This would suggest that the assumption of employing a 24 in. (61.0 cm) (x* = 5.37) depth as that at which the undisturbed ground temperature exists was appropriate. Additionally, Fig. 6a shows that the average temperature of the ground at any thermocouple for this time period is always higher than the average temperature of the air, suggesting that there is another mechanism providing energy to the ground, most likely solar radiation. This is further corroborated with data taken by Stevens [5] (Fig. 7), which show that the maximum energy derived by an operating thermoelectric device occurs in the late-morning to late-afternoon timeframe, when insolation effects are most prevalent; night radiation cooling may be seen to have a smaller, but similar effect in the pre-dawn hours.

Clearly, the two most striking items of note are in Fig. 6b:

First, not only is the optimal depth much shallower than originally thought (on the order of x* = 2.2–2.3, depending on which surface temperature profile is chosen in the finite-difference model described above—cosine, saw tooth, or step), but it is almost perfectly constant at a depth of x* ≈ 1. This is particularly noteworthy, given the air temperature swings which occurred, suggesting that convection may be a secondary effect.

Second, the energy amplification factor at this optimal location is much higher than the value of 1.04–1.07 suggested by the cosine/saw tooth/step surface temperature boundary conditions, and can vary between 1.2 < A' < 1.7 (while Fig. 6b shows swings of the range 1.2 < A' < 1.5, excursions to A' ≥ 1.7 were seen in other subsets of the data).

5. Conclusions

For the three time periods considered, the optimal location for the soil-end thermal reservoir for a thermoelectric device capitalizing on the temperature difference between the air and the ground was found to be much shallower than previous work had suggested. The optimal depth was less than half of that previously thought—not only was the optimal depth situated at approximately x* = 1, but this value remained almost constant during the timeframes investigated. Also of note was that the energy amplification factor, A', was substantially higher than values previously thought possible; instead of values on the order of A' = 1.04–1.07, it was possible for this value to range between A' = 1.2 and 1.7.

The data suggest that the radiation heat transfer mechanism plays an important role in this process, exceeding that played by convection; future work will concentrate on investigating this impact, and will most likely involve a finite-difference model which incorporates not only convective, but also radiation heat transfer at the surface. It is anticipated that not only will insolation during daylight hours be important, but that night radiation cooling will play a significant part in the phenomenon [7].

References